WIGNER'S UNITARY-ANTIUNITARY THEOREM FROM THE EYES OF JORDAN STRUCTURES

(based on a joint work with Y. Friedman)

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Up to date, much has been written about E. Wigner and U. Uhlhron theorems and their importance for physics and mathematics. True real leading experts are among the potential audience, who know well the results and even contributed to the mathematical foundations of these theory. For the sake of conciseness, let us go straight to some of the starring results. Suppose $P_1(H)$ denotes the set of all rank-one projections on a complex Hilbert space H (it can be regarded as the set of minimal projections in $B(H)$ as well as the set of all pure states in $B(H)_* = C_1(H)$).

The celebrated Uhlhorn's theorem is a generalization of Wigner's theorem where the requirement of the invariance of transition probabilities is replaced by the requirement that orthogonal vector rays are transformed into orthogonal vector rays. The concrete statement asserts that If H is a complex Hilbert space with $\dim(H) \geq 3$, then every bijective map $\Phi : P_1(H) \to P_1(H)$ which preserves orthogonality in both directions, that is,

$$pq = 0 \text{ in } P_1(H) \text{ if and only if } \Phi(p)\Phi(q) = 0,$$

is induced by a unitary or antiunitary operator on the underlying Hilbert space (see [4]).

By replacing the set of minimal projections $P_1(H)$ by the wider set $PI(H)$ of all partial isometries on H, L. Molnár proved in [3] the following attractive, and useful, result: Let H be a complex Hilbert space with $\dim(H) \geq 3$. Suppose that $\Phi : \mathcal{U}(B(H)) \to \mathcal{U}(B(H))$ is a bijective transformation which preserves the partial ordering and the orthogonality between partial isometries in both directions. If Φ is continuous (in the operator norm) at a single element of $\mathcal{U}(B(H))$ different from 0, then Φ extends to a real linear triple isomorphism. Here we consider the standard partial ordering on $PI(H)$ given by $e \leq u$ if and only if $u - e$ is a partial isometry orthogonal to e.

During this talk we shall present new results, obtained in collaboration with Y. Friedman (see [1]), showing that an extension of the previous results is possible in the case of a bijection between the lattices of tripotents of two Cartan factors and atomic JBW*-triples non-containing rank-one Cartan factors. These new result provide new models to understand the quantum models. We shall also see how the results provide new alternatives to complement recent studies by J. Hamhalter [2] proving that the set of partial isometries with its partial order and orthogonality relation is a complete Jordan invariant for von Neumann algebras.

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References

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